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THERMAL SCALE MODELING WITHOUT SIMILITUDE

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NOMENCLATURE

A, area;
B_{ij}, absorption factor, fraction of radiation leaving the node i and being absorbed by the node j;
c, specific heat [W s/kgdegK];
C, thermal conductance [W/degK];
f, "skin" scale factor s/s^*;
F, "inner" scale factor L/L^*;
g, temperature ratio T/T^*;
h, thermal contact coefficient [W/degK m^2];
L, length;
P, power dissipated in a node [W];
q, absorbed heat flux [W/m^2];
Q, absorbed heat [W];
r, ratio of absorbed heat fluxes q/q^*;
R_{ij}, radiation factor between the node i and the node j $\sigma A_i \varepsilon_i B_{ij}$;
s, skin thickness;
t, time;
T, temperature [degK];
V, volume;
x, y, directions tangential to the skin;
z, direction normal to the skin.

Greek symbols
α, absorptivity for sun- or lamp-radiation;
ε, emissivity;
λ, conductivity [W/degK m];
ρ, density [kg/m^3];
σ, constant of Stefan–Bolzmann;
φ, incident heat flux [W/m^2].

Subscripts
a, area (conduction across an interface);
i, of node i;
j, of node j;
ij, from node i to node j;
js, from node j to space;
m, material (conduction within a material);
n, normal (to the skin);
t, tangential (to the skin).

Superscripts
$*$, small model;
$'$, skin.

INTRODUCTION

WITH the development of more powerful launching vehicles the satellites or spacecraft become larger and larger. Up to now it has been necessary to let the test facilities grow in the same proportion. The laws of thermal similitude were established in the hope that it would be possible to verify or determine the thermal model† of a spacecraft after having carried out a test only on a small scale model‡ of this spacecraft, because that would permit the use of smaller test chambers.

But treating thermal scale modeling of spacecraft from

† The term "thermal model" means the thermal mathematical model, i.e. the table containing the factors in the heat balance equation [equation (1.1)].

‡ The term "scale model" means a smaller physical version of a spacecraft.

the standpoint of thermal similitude, it was found difficult in the case of temperature-preservation to find the proper materials with lower conductivities in the small model than in the prototype† [1, 2]. In the case of materials-preservation it was a limitation that the temperatures, lamp intensities and thermal contact coefficients had to be increased considerably [3, 4, 5].

The experimental difficulties have hitherto hindered the wide use of thermal model scaling for spacecraft testing [3, 5]. In order to reduce these difficulties we propose not only to use the same surface properties on the small model as on the prototype on homologous points as other authors have done, but to preserve also the materials and thermal contact coefficients and to replace the laws of thermal similitude by others. The lamp intensities and the time scales are chosen in such a way that the same temperature levels are reached in both the scale model and the prototype. The fact that the temperature gradients in the small model and prototype are different is taken into account in the calculation.

1. SCALING FACTORS AND HEAT BALANCE

Figure 1(a) shows the sketch of a spacecraft and Fig. 1(b) the sketch of the scale model. We designate with s, the skin thickness and with L, a length of the interior of the satellite; x and y are the directions tangential to the skin and z is the direction normal to the skin. An asterisk characterizes the values in the small model. A part of the satellite, as for instance the skin, which can be made of a metal sheet or a honeycomb structure, might be already relatively thin. It could then be required to use in the z direction a smaller scale factor than in the x and y directions or in other words to apply anisotropic scaling for an element of the skin. We introduce therefore $f = s/s^*$ as the scale factor of the skin and $F = L/L^*$ as the scale factor of the whole spacecraft except the skin.

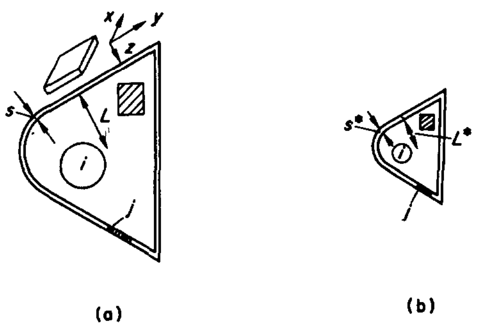

FIG. 1.

† The term "prototype" invariably refers to the latter.

If the masses are considered to be lumped together into isothermal nodes, the heat balance for the node j of Fig. 1(a) for instance becomes:

$$Q_j + P_j + \sum_i C_{ij}(T_i - T_j) + \sum_i R_{ij}(T_i^4 - T_j^4) = R_{js}T_j^4 + (\rho c V)_j \left(\frac{dT}{dt}\right)_j. \quad (1.1)$$

The heat Q_j absorbed on the outside of the node j is calculated for the case where the spacecraft is tested in a chamber simulating cold and sun and, where the reflection and emission of the chamber walls are neglected, by

$$Q_j = A_j \varphi_j \alpha_j. \quad (1.2)$$

φ is the average intensity of the lamps, which falls on the area A and α is the absorptivity for the spectrum of the lamps.

The conductance C_{ij} between the two nodes i and j is composed by conduction within the solid material (index m), which depends on the conductivity λ of the material and by conduction through the interface of two solids, which depends on the thermal contact coefficient h [index a (area)]. We can therefore write for a conductance within the spacecraft (no index), a conductance tangential to the skin (index t) and for a conductance normal to the skin (index n) [see Fig. 1(a)]:

$$C_m \sim \lambda \frac{A}{x} \sim \quad \sim \lambda L \quad (1.3a)$$

$$C_a \sim hA \sim \quad \sim hL^2 \quad (1.3b)$$

$$C_{m,t} \sim \lambda_t \frac{A_t}{x} \sim \lambda_t \frac{yz}{x} \sim \lambda_t s \quad (1.3c)$$

$$C_{a,t} \sim h_t A_t \sim h_t yz \sim h_t L s \quad (1.3d)$$

$$C_{m,n} \sim \lambda_n \frac{A_n}{z} \sim \lambda_n \frac{xy}{z} \sim \lambda_n \frac{L^2}{s} \quad (1.3e)$$

$$C_{a,n} \sim h_n A_n \sim h_n xy \sim h_n L^2. \quad (1.3f)$$

The radiation factor R_{ij} between the two nodes i and j is calculated by

$$R_{ij} = \sigma A_i \varepsilon_i B_{ij}. \quad (1.4a)$$

Because we require that the emissivities ε are conserved and identical at equivalent geometric locations, the absorption factors B_{ij} are also preserved. Therefore we can write

$$R_{ij} \sim L^2. \quad (1.4b)$$

The last term of equation (1.1) expresses the heat stored in the node j. The volume V of this term is

$$V \sim L^3, \quad (1.5a)$$

if we consider an element of the interior of the satellite. If the considered element is cut out of the skin its volume V' is

$$V' \sim sL^2. \quad (1.5b)$$

The absorbed heat Q_j, the heat radiated away to the space $R_{js}T_j^4$, the radiative heat exchange on the inside $R_{ij}(T_i^4 - T_j^4)$ and the power P_j dissipated in a node j are entirely scaled with the factor F and not f, because power is unlikely to be dissipated in a skin element. The remaining two terms of equation (1.1), the conduction $C_{ij}(T_i - T_j)$ and the storage $(\rho c V)_j(\mathrm{d}T/\mathrm{d}t)_j$ are according to the equations (1.3) and (1.5) scaled as well with F as with f.

2. THE LAWS OF THERMAL SIMILUTUDE

Introducing g as the temperature ratio T/T^* and considering the equations (1.2–1.5), we obtain from equation (1.1) for the ratio of the heat fluxes r in the prototype and the small model:

$$r = \frac{\varphi\alpha}{\varphi^*\alpha^*} = \frac{P}{P^*F^2} = \frac{\lambda g}{\lambda^* F} = \frac{hg}{h^*} = \frac{\lambda_t f g}{\lambda_t^* F^2} = \frac{h_t f g}{h_t^* F} = \frac{\lambda_n g}{\lambda_n^* f}$$

$$= \frac{h_n g}{h_n^*} = g^4 = \frac{\rho c F g t^*}{(\rho c)'^* t} = \frac{(\rho c)' f g t^*}{(\rho c)^* t}. \quad (2.1)$$

Equation (2.1) contains all the laws of thermal similitude, which require that all the terms absorption, dissipation, conduction, radiation and storage are changed with the same ratio r. The scaling laws for special cases such as temperature-preservation or materials-preservation can easily be derived from equation (2.1). Let us consider briefly these two cases.

Temperature-preservation

In this case g is unity. From equation (2.1) we see, that r should also be unity. We find further that for instance the conductivity $\lambda^* = \lambda/F$ and the time $t^* = t(\rho c)^*/\rho c F = t(\rho c)'^*/(\rho c)' f$ have to be reduced with the scale factor.

Materials-preservation

In this case we have $\lambda = \lambda^*$ and equation (2.1) gives $r = g/F = g^4$, which gives for the temperature ratio $g = T/T^* = (1/F)^{\frac{1}{3}}$ and for the ratio of the heat fluxes $r = \varphi\alpha/\varphi^*\alpha^* = (1/F)^{\frac{4}{3}}$. Further we obtain $h^* = h/g^3 = hF$. So the temperatures T^*, the lamp intensities φ^* and the thermal contact coefficients h^* have to be increased considerably. The time $t^* = t\rho c^*/(\rho c)F^2$ has to be reduced with the second power of the scale factor.

3. THE LAWS FOR SCALING WITHOUT SIMILITUDE

From equation (1.4b) we obtain for the radiation factors between the nodes and to the space

$$R_{ij} = R_{ij}^* F^2 \quad (3.1a)$$

$$R_{js} = R_{js}^* F^2. \quad (3.1b)$$

By equation (1.5a) the heat capacity of the inside of the prototype is determined by

$$(\rho c V)_j = (\rho c V)_j^* F^3 \quad (3.2a)$$

and the heat capacity of a skin-node by equation (1.5b) as

$$(\rho c V)_j' = (\rho c V)_j'^* F^2 f. \quad (3.2b)$$

The conductance C_{ij} between the two nodes i and j is calculated by

$$C_{ij} = \frac{C_m C_a}{C_m + C_a}. \quad (3.3)$$

The conductance C_m due to solid conduction in the material is composed of conduction within the node i and of conduction within the node j. From the equations (1.3) we obtain:

$$C_m = F C_m^* \quad (3.4a)$$

$$C_a = F^2 C_a^* \quad (3.4b)$$

$$C_{m,t} = f C_{m,t}^* \quad (3.4c)$$

$$C_{a,t} = F f C_{a,t}^* \quad (3.4d)$$

$$C_{m,n} = \frac{F^2}{f} C_{m,n}^* \quad (3.4e)$$

$$C_{a,n} = F^2 C_{a,n}^*. \quad (3.4f)$$

Introducing the equations (3.4) into equation (3.3) it follows:

$$C_{ij} = \frac{F^2 C_m^* C_a^*}{C_m^* + F C_a^*} \quad (3.5a)$$

$$C_{ij,t} = \frac{F f C_{m,t}^* C_{a,t}^*}{C_{m,t}^* + F C_{a,t}^*} \quad (3.5b)$$

$$C_{ij,n} = \frac{F^2 C_{m,n}^* C_{a,n}^*}{C_{m,n}^* + f C_{a,n}^*}. \quad (3.5c)$$

The conductances within the satellite C_{ij}, tangential to the skin $C_{ij,t}$ and normal to the skin $C_{ij,n}$ of the prototype can thus be calculated, when not only are the total conductances in the small model determined by a test, but also the contributions of conduction within the material (C_m) and across the joints (C_a^*) are known. If the temperatures are measured only at the centres of the nodes i and j only the total conductance C_{ij} of the scale model can be determined. In this case one of the conductances C_m^* or C_a^* in the equations (3.5) would have to be calculated theoretically. If the temperature drop across the interface is small in comparison to the temperature drop between the centres of the nodes, we would have $C_a^* \gg C_m^*$ and equation (3.5a) would give $C_{ij} \approx F C_m^*$. If the thermal resistance across the joint dominates, we have $C_m^* \gg C_a^*$ and equation (3.5a) gives $C_{ij} \approx F^2 C_a^*$. Similar laws can be derived from the equations (3.5b) and (3.5c).

The interface heat transfer depends on the surface roughness and waviness, the pressure, the materials of the two surfaces and any interstitial fluid. As much as possible has to be done for the case in which the interface thermal resistance is not negligible to ensure that these parameters are identical in the small model and the prototype. But this difficulty exists also to a still larger extent in the case of scaling with thermal similitude, where either the materials are not the same or the thermal contact coefficients have to be increased artificially.

It might be remembered that the view factors are difficult to determine for irregular geometries. Even if they are calculated exactly by numerical integration using a computer and converted into the radiation factors R_{ij}, R_{js}, these factors would not be the same as those obtained by a test. This is due to the fact that the number of nodes is always too low.

Practically we determine at first the thermal model of the scale model [i.e. the factors C_{ij}, R_{ij}, R_{js} and $(\rho cV)_j$ in equation (1.1)] as if this model would be the real spacecraft [6]. Then we calculate with the equations (3.1–3.5) the thermal model of the prototype. In order to have in the scale model and in the prototype the same temperature levels, the lamp intensities φ can be at the same order of magnitude, but the absorbed heats Q and dissipated heats P have to be reduced with F^2.

4. EXAMPLES OF CALCULATION

Let us consider models with only two nodes j and i as in Fig. 2. The node j can be considered as a circular plate and the node i as a segment of a spherical shell. But the conductances are calculated by supposing that we have a two-dimensional problem and that the Figs. 2 are a cut through infinite long cylinders, from which we consider in the case of model 1, a length of 1 m and in the case of model 2, a length of 0·1 m.

Table 1 contains the thermal models for five cases. We have models of three different geometries (model 1, 2 and 3) and two different materials (steel and aluminium). Between the models 1 and 2 the scale factors are $F = 10$ and $f = 5$. Between the models 1 and 3 we have $F = 10$ and $f = 10$. Here only a conductance tangential to the skin exists. All surfaces are considered to be perfectly black.

Table 2 contains for the case of one solar constant falling normally on the node j the equilibrium temperatures and the heat exchange by conduction and radiation between the nodes j and i as obtained by equation (1.1). In the cases 3 and 4 T_j and T_i are identical, because the laws of the temperature-preservation technique are fulfilled between the model 3 (made of steel) and the model 1 (made of aluminium). In these two cases (3 and 4) the ratio of the conductive heat transfer to the radiative heat transfer is also the same. Otherwise this ratio increases if the model

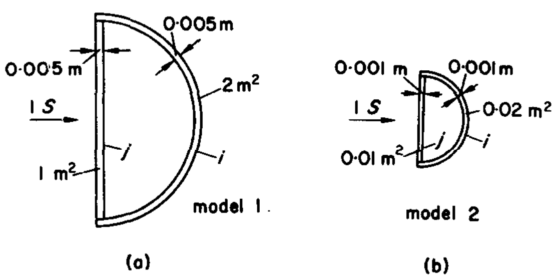

FIG. 2.

Table 1

Case No.	Model	Material	L [m]	s [m]	λ [W/degK m]	C_m [W/degK]	h_t [W/degK m²]	C_a [W/degK]	C_{ij} [W/degK]
1	1	steel	1	0·005	21	0·07	500	2·5	0·068
2	2	steel	0·1	0·001	21	0·014	500	0·05	0·01093
3	3	steel	0·1	0·0005	21	0·007	500	0·025	0·00546
4	1	aluminium	1	0·005	210	0·7	500	2·5	0·546
5	2	aluminium	0·1	0·001	210	0·14	500	0·05	0·0369

Case No.	$R_{ij} = R_{js}$ [W/degK⁴]	R_{is} [W/degK⁴]	ρ [kg/m³]	c [kcal/kg degK]	V_j [m³]	$(\rho cV)_j$ [J/degK]	$(\rho cV)_i$ [J/degK]
1	$5{\cdot}69 \cdot 10^{-8}$	$11{\cdot}38 \cdot 10^{-8}$	7900	0·12	0·005	19900	39800
2	0·0569	0·1138	7900	0·12	0·00001	39·8	79·6
3	0·0569	0·1138	7900	0·2	0·000005	19·9	39·8
4	5·69	11·38	2700	0·225	0·005	12700	25400
5	0·0569	0·1138	2700	0·225	0·00001	25·4	50·8

Table 2

Case No.	T_j [°K]	T_i [°K]	Conduction [W]	Radiation [W]	$A_j\varphi_j\alpha_j$ [W]
1	348·6	265·3	5·66	556	1400
2	345·3	268·9	0·835	5·099	14
3	347·0	267·1	0·436	5·338	14
4	347·0	267·1	43·6	533·8	1400
5	338·7	275·5	2·33	4·2	14

Table 3

t [s]	Case No. 1		Case No. 4	
	T_j [°K]	T_i [°K]	T_j [°K]	T_i [°K]
120	340·7	265·2	335·0	266·8
240	333·5	264·8	324·7	266·1
360	327·0	264·4	315·9	265·0
480	321·2	263·8	308·1	263·7
600	315·8	263·0	301·3	262·3

t [s]	Case No. 2		Case No. 3		Case No. 5	
	T_j [°K]	T_i [°K]	T_j [°K]	T_i [°K]	T_j [°K]	T_i [°K]
60	327·1	268·2	314·7	264·7	312·6	273·6
120	312·8	266·5	293·4	260·1	294·7	269·6
180	301·3	264·2	277·9	254·7	281·5	264·8
240	291·7	261·7	266·0	249·2	271·1	259·8
300	283·5	268·9	256·2	243·2	262·7	254·8

size decreases and the materials are conserved. The examples show also that the temperature gradients are reduced in the small models, because the heat transfer by conduction becomes more important with respect to the radiative heat transfer. So the effect of temperature-gradient reduction is more important in the case where the material is aluminium (cases 4 and 5) than in the case where the material is steel (cases 1 and 2).

Table 3 gives the temperatures as a function of time, when the models are cooled down from their equilibrium temperatures in a perfectly black space as calculated by a computer. As we expect, the small models cool down more quickly.

The thermal models in Table 1 had been calculated for each of the five cases considered independently, But because the relations (3.1), (3.2b), (3.4c), (3.4d) and (3.5b) are fulfilled between the cases in which the materials are conserved, these relations could be used to calculate the factors in the heat balance equation for the large models (cases 1 and 4), if a test would have been made only for the small models (cases 2, 3 and 5). For the general case of a much more complicated spacecraft all the equations (3.1–3.5) will be needed, but the general procedure is the same.

CONCLUSION

Equations are given to convert the factors in the heat balance equation determined by a test for a small scale model into the corresponding factors of the real spacecraft. Because this conversion ratio is not constant for the different terms (conduction, radiation and storage) a coincident family of temperature curves can not be obtained and more theoretical work is required. However not much more work is required, because a detailed thermal analysis is needed also in the case where the laws of similitude are fulfilled or if the test is made on the large prototype. On the other hand the test would be much simpler, because one would not be obliged to think about the questions of how to find the right materials or how to increase the lamp intensities or the thermal contact coefficients.

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A MODEL RATE EQUATION FOR TRANSIENT THERMAL CONDUCTION

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NOMENCLATURE

$\vec{a}$,	acceleration of an electron or molecule between 2 collisions;
c,	specific heat;
c_p,	specific heat at constant pressure;
c_v,	specific heat at constant volume;
c_m,	velocity of momentum propagation;
c_w,	velocity of heat propagation;
E,	energy of a molecule or electron;
f,	distribution function;
f_0,	equilibrium distribution function;
K,	thermal conductivity;
k,	Boltzmann's constant;
m,	mass of an electron or molecule;
n,	number of free electrons or molecules;
P,	per unit volume;
p,	pressure;
$\vec{q}$,	heat flux vector;
T,	temperature;
t,	time;
u,	velocity in x direction;
v,	velocity vector;
w,	velocity in z direction;
x, y, z,	coordinate axes.

Greek symbols

α,	thermal diffusivity;
ϕ,	as defined by equation (12);
γ,	c_p/c_v;
μ,	viscosity;
ρ,	density;
τ,	shear stress;
τ_c,	relaxation time.

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1. INTRODUCTION

AT THE present time the analytical treatment of diffusional type of transfer processes is restricted to the domain of the validity of the phenomenological relation given in the form

$$J_i = L_{ij} X_j \tag{1}$$

where J_i represents fluxes and X_j the thermodynamic forces.

Strictly speaking, equation (1) should be applied only to low rate steady-state transfer processes, and all L_{ij} are defined in this manner. However, in practice the validity of relation (1) is successfully extended to the unsteady processes without altering the values of L_{ij}. Phyiscally, there must be a time scale where the validity of (1) is violated. Several investigators [13, 14] having this in mind, tried experimentally, by applying relation (1) to transient heat conduction to show that the heat flow is not only proportional to the temperature gradient, but also rate dependent.

The results of these experiments are not conclusive: the authors in [14] had indicated the possibility of a significant